Pour s'entraîner :

$\mathcal{L}[t^5 \cdot U(t)] =$

$\mathcal{L}[\cos(2t)] =$

$\mathcal{L}[e^{-t} \sin(3t) \cdot U(t)] =$

$\mathcal{L}[3t^2 - 5t + 1] =$

$\mathcal{L}[e^{t-2} \cdot U(t)] =$

$\mathcal{L}[e^{t-2} \cdot U(t-2)] =$

$\mathcal{L}[e^{t} \cdot U(t-2)] =$

$\mathcal{L}[t + 1 - \sin t] =$

Pour s'entraîner :

$$\mathcal{L}[t^5 \cdot U(t)] = \frac{5!}{p^6} = \frac{1\times2\times3\times4\times5}{p^6} = \frac{120}{p^6}$$

$$\mathcal{L}[\cos(2t)] = \frac{p}{p^2+4}$$

$$\mathcal{L}[e^{-t}\sin(3t)\cdot U(t)] = \frac{p+1}{(p+1)^2+9} = \frac{p+1}{p^2+2p+10}$$

$$\mathcal{L}[3t^2-5t+1] = \frac{6}{p^3} - \frac{5}{p^2} + \frac{1}{p}$$

$$\mathcal{L}[e^{t-2}\cdot U(t)] = \mathcal{L}[e^{-2}\cdot e^{t}] = e^{-2}\cdot\mathcal{L}[e^t] = \frac{e^{-2}}{p-1}$$

$$\mathcal{L}[e^{t-2}\cdot U(t-2)] = e^{-2p}\cdot\mathcal{L}[e^t\cdot U(t)] = \frac{e^{-2p}}{p-1}$$

$$\mathcal{L}[e^{t}\cdot U(t-2)] = e^{-2p}\cdot\mathcal{L}[e^{t+2}\cdot U(t)] = e^{-2p}\cdot\mathcal{L}[e^{2}\cdot e^{t}] = e^{-2p+2}\cdot\frac{1}{p-1}$$

$$\mathcal{L}[t+1-\sin t] = \frac{1}{p^2} + \frac{1}{p} - \frac{1}{p^2+1}$$

Pour s'entraîner :

$$\mathcal{L}^{-1}\left[\frac{3}{p^5}\right] =$$

$$\mathcal{L}^{-1}\left[\frac{p+1}{p^2+4}\right] =$$

$$\mathcal{L}^{-1}\left[e^{-3p}\cdot\frac{p+1}{p^2+4}\right] =$$

$$\mathcal{L}^{-1}\left[\frac{1}{p^2-3p+2}\right] =$$

$$\mathcal{L}^{-1}\left[\frac{1}{(p^2+1)(p-5)^2}\right] =$$

$$\mathcal{L}^{-1}\left[\frac{e^{-p}}{p}\right] =$$

Pour s'entraîner :

$$\mathcal{L}^{-1}\left[\frac{3}{p^5}\right] = 3\,\mathcal{L}^{-1}\left[\frac{1}{4!}\frac{1\times 4!}{p^5}\right] = \frac{3}{24}\,t^4.U(t)$$

$$\mathcal{L}^{-1}\left[\frac{p+1}{p^2+4}\right] = \mathcal{L}^{-1}\left[\frac{p}{p^2+4}\right] + \mathcal{L}^{-1}\left[\frac{1}{2}\frac{1\times 2}{p^2+4}\right] = \left(\cos(2t) + \frac{1}{2}\sin(2t)\right).U(t)$$

$$\mathcal{L}^{-1}\left[\underbrace{e^{-3p}}_{\text{retard de }3}.\frac{p+1}{p^2+4}\right] = f(t-3) \text{ où } f(t) = \mathcal{L}^{-1}\left[\frac{p+1}{p^2+4}\right]$$

$$= \left(\cos(2t-6) + \frac{1}{2}\sin(2t-6)\right).U(t-3)$$

$$\mathcal{L}^{-1}\left[\frac{1}{p^2-3p+2}\right] = \mathcal{L}^{-1}\left[\frac{1}{(p-1)(p-2)}\right] = \mathcal{L}^{-1}\left[\frac{-1}{p-1}+\frac{1}{p-2}\right] = -\mathcal{L}^{-1}\left[\frac{1}{p-1}\right] + \mathcal{L}^{-1}\left[\frac{1}{p-2}\right]$$

$$= -e^t + e^{2t} = e^t\left(e^t - 1\right)$$

$$f(t) = \mathcal{L}^{-1}\left[\frac{1}{(p^2+1)(p-5)^2}\right] = \mathcal{L}^{-1}\left[\frac{ap+b}{p^2+1} + \frac{c}{(p-5)^2} + \frac{d}{p-5}\right] =$$

$$\bullet\ ai+b = \frac{1}{(i-5)^2} = \frac{1}{25-10i-1} = \frac{1}{24-10i} = \frac{1}{2}\,\frac{1}{12-5i}\times\frac{12+5i}{12+5i} = \frac{12+5i}{2(169)}$$

$$\Leftrightarrow a = \frac{5}{338} \text{ et } b = \frac{12}{338}\ ;\quad \bullet\ c = \frac{1}{26}\ ;\quad \bullet\ \lim_{p\to+\infty} pF(p) = 0 = a+d \Leftrightarrow d = -\frac{5}{338}.$$

$$f(t) = \left(\frac{5}{338}\cos(t) + \frac{12}{338}\sin(t) + \frac{1}{26}e^{5t}.t - \frac{5}{338}.e^{5t}\right).U(t).$$